Sr. No. 9019

M.Sc. (Maths), Third Semester Examination, Dec 2016

Mathematical Statistics

Time: 3 Hours | Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) Give classical definition of probability. A bag contains 4 white, 5 red and 6 green balls. Three balls are drawn at random. What is the probability that a white, a red and a green ball are drawn?
   (b) State and prove addition theorem of probabilities.
2. (a) State and prove Bayes' theorem on probability.
   (b) A bag contains 6 white and 9 black balls. Four balls are drawn at a time. Find the probability for the first draw to give 4 white and the second to give 4 black balls in each of the following cases:
   i) The balls are replaced before the second draw.
   ii) The balls are not replaced before the second draw.

Unit – II

3. (a) Explain the following: Sample Space, distribution function, Discrete and Continuous random variables alongwith one example for each.
   (b) A random variable X has the following probability distribution:

X:	0	1	2	3	4	5	6	7	8
f(x):	K	3K	5K	7K	9K	11K	13K	15K	17K

Find the value of K; $P(X<3)$; $P(X\geq 3)$; $P(0<X<5)$. Also obtain mean of the distribution.

4. (a) Given the following bivariate probability distribution, obtain (i) marginal distributions of X and Y. (ii) Conditional distribution of X given Y=2.

Y \ X	-1	0	1
0	1/15	2/15	1/15
1	3/15	2/15	1/15
2	2/15	1/15	2/15

   (b) Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.
   (c) Prove that the moment generating function of the sum of a number of independent random variables is rqual to the product of their respective moment generating functions.

Unit – III

5. (a) Define Binomial distribution and prove the following relation for this distribution:
   $$\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d}{dp}\mu_r\right]$$
   (b) Find moment generating and cumulant generating functions for Poisson's distribution. Also show that all cumulants are equal for Poisson's distribution.
6. (a) Prove that for a Normal distribution:
   $$\mu_{2n+1} = 0 \; and \; \mu_{2n} = 1.3.5 \ldots\ldots (2n-1)\sigma^{2n}$$
   (b) Define exponential distribution. Obtain its mean, variance and moment generating function.

P.T.O.

Unit – IV

7. (a) What do you understand by estimation?
   (b) Write short notes on the following:
   i) Simple and Composite Hypothesis
   ii) Two types of errors.
   iii) Standard error of estimate.

8. (a) A dice is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the dice cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies.
   (b) A sample of 900 members has a mean 3.4 cms and S.D. 2.61 cms. Is the sample from a large population of mean 3.25cms and S.D. 2.61 cms? If the population is normal and its mean is unknown, find the 95% and 98% fiducial limits of true mean.

Unit – V

9.
   a) Define mutually exclusive events and equally likely events.
   b) Find the probability of throwing a number greater than 2 with an ordinary dice?
   c) Define geometric distribution.
   d) Find the expectation of the number on a dice when thrown.
   e) Explain moment generating function. Why is it called so?
   f) Obtain M.G.F. for a Normal distribution.
   g) Define parameter & statistic.
   h) State Central limit theorem.

Sr. No. 9020

M.Sc. (Maths), Third Semester Examination, Dec 2016
Integral Equations & Calculus of Variations

Time: 3 Hours | Max. Marks: 80

Note: Attempt one question from each of four units. Unit V is compulsory.

Unit – I

1. (a) Define integral equations. Also describe the various types of linear integral equations. 8

   (b) Reduce the IVP 8

   y''(x)+A(x)y'(x)+B(x)y(x)=f(x)'

   $y(a) = q_0, y'(a) = q_1, a \le x \le b$ to the Volteera IE.

2. (a) Find the Newmann series for the solution of the IE $y(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2}\, y(t)dt$ 8

   (b) Solve the generalized Abel's IE $f(x) = \int_0^x \frac{y(t)dt}{(x-t)^\alpha}, 0 < \alpha < 1$ 8

Unit – II

3. (a) Explain the method of successive approximation for the solution of Fredholm IE. 8

   (b) Solve $y(x) = f(x) + \lambda \int_0^1 xe^t y(t)dt$ by the method of Fredholm determinants 8

4. (a) Show that the IE $y(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)y(t)dt$ possesses no solution for $f(x) = x$. 8

   (b) Find the approximate solution to the Fredholm IE $y(x) = x + \int_{-1}^{1} e^{xt}\, y(t)dt$ by considering only the first two terms of e^{xt}. 8

Unit – III

5. (a) Solve $y'' = -x, y(0) = 0, y(1) = 0$ using Green's function and verify the answer. 8

   (b) Reduce the BVP $y'' + xy = 1, y(0) = 0, y(l) = 1$ into IE using Green's function. 8

6. (a) State and prove Hilbert Schmidt theorem. 8

   (b) Find the e-values and e-functions of the homogeneous equation $y(x) = \lambda \int_0^1 K(x,t)\, y(t)$ with symmetric keenel $K(x,t) = \begin{cases} x(1-t), & x < t \\ t(1-x), & x > t \end{cases}$ 8

Unit – IV

7. (a) Find the curve passing through (x_0, y_0) and (x_1, y_1) which generates the surface of minimum area when rotated about x-axes. 8

   (b) Find the extremals of the functional $\int_{-a}^{a} (Py + \frac{1}{2}uy''^2)dx$, which satisfies the boundary conditions $y(-a) = 0, y(a) = 0, y'(-a) = 0, y'(a) = 0$ 8

8. (a) Prove that the sphere is a solid figure of revolution which for a given surface area has a maximum volume. 8

   (b) Find the geodesies on a right circuler cylinder of radius a. 8

Unit – V

9. 16

   a) Mention the different types of keenels.
   b) State Fredholm first theorem.
   c) Explain approximate method for the solution of Fredholm IE.
   d) State four basic properties of Green's function.
   e) Reduce VIE of first kind $x = \int_0^x 3^{x-t} y(t)dt$ into second kind.
   f) State any two properties of e-values and e-functions of a symmetric keenel.
   g) Distinguish between functions and functional.
   h) Define Isoperimetric problems. Also provide suitable examples.

Sr. No. 9018

M.Sc. (Maths), Third Semester Examination, Dec 2016
Number Theory

Time: 3 Hours   Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) Find all solutions of 999x-49y=5000. 8
   (b) If u and v are relatively prime positive integers whose product uv is a perfect square, then u and v are both perfect squares. 8
2. (a) Prove that the equation $x^4 + y^4 = z^4$ has no solution in positive integers. 8
   (b) Prove that the cubic cureve $x^3 + y^3 = 9$ contains infinitely many rational points. 8

Unit – II

3. (a) if $\frac{a}{b}$ and $\frac{a^1}{b^1}$ are consecutive fractions in any row, then among all rational fractions with values between these two, $\frac{a+a^1}{b+b^1}$ is the unique fraction with smallest denominator. 8
   (b) State and prove Blichfeldt's principle. 8
4. State and prove Lagrange's four square theorem. 16

Unit – III

5. (a) For any positive real number x, $< a_0, a_1, \dots, a_{n-1}, x >= \frac{xh_{n-1}+h_{n-2}}{xk_{n-1}+k_{n-2}}$ 8
   (b) The convergents ${}^{h_n}/_{k_n}$ are successively closer to ξ, where ξ is an irrational number. 8
6. Prove that the continued fraction expansion of the real quadratic irrational number ξ is purely periodic iff $\xi > 1$ and $-1 < \xi' < 0$, where ξ' denotes the conjugate of ξ. 16

Unit – IV

7. (a) for $n \geq 1$, we have $p^d(n) = p^0(n)$, where $p^d(n)$ is number of partitions of n into distinct parts. 8
   (b) Suppose $0 \leq x < 1$ and let $\phi_m(x) = \prod_{n=1}^{m}(1 - x^n)$. Then prove that $\sum_{n=0}^{\infty} p_m(n)x^n$ converges and $\sum_{n=0}^{\infty} p_m(n)x^n = \frac{1}{\phi_m(x)}$. 8
8. (a) Prove that p(5m+4)≡0(mod5). 8
   (b) For $0 \leq x < 1$, prove that $x\phi(x)^4 = \sum_{m=1}^{\infty} b_m x^m$, where b_m are integers and $b_m \equiv 0(mod5)$ if $m \equiv 0(mod5)$. 8

Unit – V

9. This question is compulsory: (2 marks each)
   a) Define rational points.
   b) Define ternary quadratic form.
   c) State Hurwitz theorem.
   d) Define Farey sequence.
   e) Define periodic continued fraction.
   f) Define Pell's equation.
   g) State Euler's formula.
   h) State Jacobi's formula.

Sr. No. 9017

M.Sc. (Maths), Third Semester Examination, Dec 2016

Partial Differential Equations and Mechanics

Time: 3 Hours — Max. Marks: 80

Note: Attempt five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. Using method of separation of variables, find solution of one-dimensional heat equation.
2. Find a general solution of Laplace equation in cylindrical co-ordinates, using the method of separation of variables.

Unit – II

3. Obtain solution of two-dimensional wave equation by the method of separtion of variables.
4. A string of length l has its ends $x = 0$ and $x = l$ fixed. It is released from rest in the postion $y = \frac{4\lambda x(1-x)}{l^2}$. Find an expression for the displacement of the string at any subsequent time.

Unit – III

5. (a) A rigid body S has a spin $\underset{w}{\rightarrow}$ and a particle A of S has velocity $\underset{v}{\rightarrow}$. Show that every particle P of S with velocity vector parallel to $\underset{w}{\rightarrow}$ lies on the line:

   $\underset{AP}{\rightarrow} = (\underset{w}{\rightarrow} x \underset{v}{\rightarrow})|w^2 + \mu \underset{w}{\rightarrow}$, *where μ being an arbitrary scalar and* $w = |\underset{w}{\rightarrow}|$.

   (b) Obtain the formula for moving axes:

   $\frac{d\vec{F}}{dt} = \frac{\partial \vec{F}}{\partial t} + \underset{w}{\rightarrow} x \underset{F}{\rightarrow}$, where $\underset{F}{\rightarrow}$ is any differentiable vector function.

6. (a) Discuss the motion of a uniform solid circular cylinder down a rough inclined plane.

   (b) A uniform rod AB of mass M and length 2a lies at rest on a smooth horizontal table. An impulse J is applied at A in the plane of the table and perpendicular to the rod. Determine the velocity of the centroid and the angular velocity of the rod.

Unit – IV

7. (a) Define moments and products of Inertia. Obtain the M.I. of the distribution about an axis through O having d.c.'s < l,mn> in terms of these d.c.'s and A,B,C,D,E,F.

   (b) Find the directions of principal axes at one corner of a uniform rectangular lamina of demensions 2ax2b.

8. (a) Define equimomental systems. Desire necessary and sufficient conditions for two systems to be equimomental.

   (b) A uniform solid rectangular block is of mass M and dimensions 2ax2bx2c. Find the equation of the momental ellipsoid for a corner O of the block, referred to the edges through O as co-ordinate axes.

Unit – V

9. Compulsory question.

   a) Write the relation between Cartesian (x,y,z) and spherical co-ordinates (r,θ, $\emptyset$).

   b) Write Laplace equation in spherical co-ordinates.

   c) Solve BVP. $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ with $u(0, y) = 8e^{-3y}$ by the method of separation of variables.

   d) What do you mean by moving co-ordinate system

   e) Explain momental ellipsoid.

   f) State perpendicular axis theorem.

   g) Write two-dimensional heat equation in cartesion co-ordinates.

   h) Define Principal axes and principal Moments of Incrtia.

Sr. No. 6174-A

M.Sc. (Maths), Third Semester Examination, Dec 2016

Mathematical Statistics

Time: 3 Hours

Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) A random variobale X has the following probability distribution:

Value of X, x	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

i. Determine the values of a.

ii. Find P(X<3), P(X≥3), P(0<X<5)

iii. What is the smallest value of x for which $P(X \leq x) > \frac{1}{2}$?

iv. Find out the distribution of X?

(b) A box contains 2^n tickets among which n_{C_i} tickets bear the number $i, i = 0,1,2,\ldots,n$. A group of m tickets is drawn. What is the expectation of the sum of their numbers?

2. (a) A man with n keys wants to open his door and tries the keys independently and at random. Find the mean and variance of the number of trials required to open the door (i) if unsuccessful keys are not eliminated from further selection and (ii) if they are.

(b) Show that co-efficient of correlation 'r' is independent of a change of scale and origin of the variable. Also prove that for two independent variables r=0. Also calculate the correlation co-efficient between the variables X& Y:

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

Unit – II

3. (a) Let X is a random variable following binomial distribution with mean 2.4 and variance 1.44. Find P(X≥5).

(b) State and prove the recurrence relation for the moments of Poisson distribution.

4. (a) Explain the t-test for testing the significance of an observed sample correlation. A random sample of 27 pairs of observations from a normal population gives a correlation co-efficient of 0.42. Is it likely that the variables in the populations are uncorrelated?

(b) The time taken by workers in performing a job by method I and method II is given below:

Mehtod I:	20	16	26	27	23	22	
MethodII:	27	33	42	35	32	34	38

Do the data show that the variances of time distribution from population from which these samples are drawn do not differ significantly?

Unit – III

5. (a) Define moment generating function of r.u. Hence or otherwise find m.g.f. of Y=aX+b.

(b) Discuss three properties of moment generating function (m.g.f.)

6. (a) If the moments of variable 'X' are defined by $E(X^r)=0.6$, r=1,2,3,4,..., show that P(X=0)=0.4, P(X=1)=0.6, P(X≥2)=0.

P.T.O.

(b) Give one observation from a population with p.d.f.:

$f(x, Q) = \frac{2}{Q^2}(Q - x), 0 \leq x \leq Q$. obtain 100(1−∝)% confidence interval for Q.

Unit – IV

7. (a) Define & explain the following with example:

   i) Point estimation ii) Sufficient Statistic

   (b) State and prove Rao Balckwell theorem.

8. (a) What is an unbiased estimator? If $X_1, X_2, X_3, \ldots, X_n$ is a random sample from a normal population N(μ,1), then show that $t = \frac{1}{n}\sum x_i^2$ is an unbiased estimator of $\mu^2 + 1$.

   (b) Define efficiency. Prove that sample mean is more efficient than the sample median for normal samples. Also find the efficiency of the sample median.

Unit – V

9.

a) Find mean and variance of Binomial distribution.

b) If Probability density function of a random variable is given by

$f(n) \begin{cases} k(1 - x^2) \\ 0 \end{cases}$ $if\ 0 \leq x \leq 1$ elsewhere Find the value of k.

c) Write p.d.f. of normal function.

d) Explain the term unbiasedness of an estimator. Give suitable example.

e) State Chebyshev's inequality and give its uses.

f) Write down control moments of a normal distribution.

g) Write property of sufficient estimators.

h) Define correlation. Discuss its significance.

Sr. No. 9019

M.Sc. (Maths), Third Semester Examination, Dec 2016

Mathematical Statistics

me: 3 Hours Max. Marks: 80

ote: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

(a) Give classical definition of probability. A bag contains 4 white, 5 red and 6 green balls. Three balls are drawn at random. What is the probability that a white, a red and a green ball are drawn?

(b) State and prove addition theorem of probabilities.

(a) State and prove Bayes' theorem on probability.

(b) A bag contains 6 white and 9 black balls. Four balls are drawn at a time. Find the probability for the first draw to give 4 white and the second to give 4 black balls in each of the following cases:

i) The balls are replaced before the second draw.

ii) The balls are not replaced before the second draw.

Unit – II

(a) Explain the following: Sample Space, distribution function, Discrete and Continuous random variables alongwith one example for each.

(b) A random variable X has the following probability distribution:

X:	0	1	2	3	4	5	6	7	8
f(x):	K	3K	5K	7K	9K	11K	13K	15K	17K

Find the value of K; $P(X<3)$; $P(X\geq 3)$; $P(0<X<5)$. Also obtain mean of the distribution.

(a) Given the following bivariate probability distribution, obtain (i) marginal distributions of X and Y. (ii) Conditional distribution of X given Y=2.

Y \ X	-1	0	1
0	1/15	2/15	1/15
1	3/15	2/15	1/15
2	2/15	1/15	2/15

(b) Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.

(c) Prove that the moment generating function of the sum of a number of independent random variables is rqual to the product of their respective moment generating functions.

Unit – III

(a) Define Binomial distribution and prove the following relation for this distribution:

$$\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d}{dp}\mu_r\right]$$

(b) Find moment generating and cumulant generating functions for Poisson's distribution. Also show that all cumulants are equal for Poisson's distribution.

(a) Prove that for a Normal distribution:

$$\mu_{2n+1} = 0 \text{ and } \mu_{2n} = 1.3.5 \ldots..(2n-1)\sigma^{2n}$$

(b) Define exponential distribution. Obtain its mean, variance and moment generating function.

P.T.O.

Unit – IV

7. (a) What do you understand by estimation?

   (b) Write short notes on the following:

   i) Simple and Composite Hypothesis

   ii) Two types of errors.

   iii) Standard error of estimate.

8. (a) A dice is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the dice cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies.

   (b) A sample of 900 members has a mean 3.4 cms and S.D. 2.61 cms. Is the sample from a large population of mean 3.25cms and S.D. 2.61 cms? If the population is normal and its mean is unknown, find the 95% and 98% fiducial limits of true mean.

Unit – V

9.

a) Define mutually exclusive events and equally likely events.

b) Find the probability of throwing a number greater than 2 with an ordinary dice?

c) Define geometric distribution.

d) Find the expectation of the number on a dice when thrown.

e) Explain moment generating function. Why is it called so?

f) Obtain M.G.F. for a Normal distribution.

g) Define parameter & statistic.

h) State Central limit theorem.

Sr. No. 9016

M.Sc. (Maths), Third Semester Examination, Dec 2016

Functional Analysis – I

Time: 3 Hours Max. Marks: 80

Note: Attempt five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) Let p be a real number such that $1 \leq p < \infty$ and let ℓ_p denote the space of all sequences $x = < x_1, x_2, ... >$ of scalars such that $\sum_{n=1}^{\infty} |x_n|^p < \infty$. Prove that ℓ_p is a Banach space under the norm $||x||_p = (\sum_{i=1}^{\infty} |x_i|^p)^{\frac{1}{p}}$. 10

   (b) Prove that every convergent sequence is a Cauchy sequence in a normed linear space. 6

2. (a) State and prove Riesz Fisher theorem. 8

   (b) Prove that the linear space C[0,1] of all real valued continuous functions on [0,1] is incomplete normed linear space under the norm $||f|| = \int_0^1 |f(x)| dx, f \in c[0,1]$. 8

Unit – II

3. (a) Prove that every finite dimensional subspace Y of a normed linear space X is closed in X. Give an example to show that infinite dimensional subspaces need not be closed. 8

   (b) Let N and N' be normed linear spaces and T a linear transformation of N into N'. Then prove that T is bounded if and only if the image T(S) of the closed unit sphere S={x:||x||≤1} in N is bounded set in N'. 8

4. State and prve Hahn Banach extension theorem. Also give two applications of this result. 16

Unit – III

5. (a) What is the natural imbedding of N into N**? Explain it. 8

   (b) Prove that if the spaces X and Y are each Banach spaces then B(X,Y) is complete in the strong sense. 8

6. (a) Prove that a linear transformation is closed if and only if its graph is a closed subspace. 8

   (b) Prove that in a normed linear space X, every weak Cauchy sequence is bounded. 8

Unit – IV

7. (a) Suppose A:X→Y where X and Y are normed linear spaces. If X is fininte dimensional, A is bounded. 8

   (b) In a finite dimensional space, the notion of weak and strong convergence are equivalent. 8

P.T.O.

8. (a) Let <T_n> be a sequence of compact linear operators from a normed space X into a Banach space Y. If <T_n> is uniformly operator convergent, say $||T_n-T||\rightarrow 0$, then the limit operator T is compact. 8

(b) Let X and Y be normed spaces. Then 8

a) Every compact linear operator $T:X\rightarrow Y$ is bounded, hence continuous.

b) If $\dim X=\infty$, the identity operator $I:X\rightarrow X$ (which is continuous) is not compact.

Unit – V

9. This questin is compulsory: (2 marks each)

a) Give example of a space which is complete as well as incomplete normed linear space.

b) Define Hilbert co-ordinate space.

c) State Riesz Representaiton Theorem for Bounded Linear Functional on.L^p

d) Define Second Conjugate space.

e) Give two applications of Uniform Boundedness principle.

f) State closed Graph theorem.

g) Define completely continuous linear operator.

h) What are projections on Banach spaces?